439. Proposed by CHARLES N. SCHMALL, New York City.

Show that the areas of any two triangles circumscribed about the same circle are in the same ratio as their perimeters.

440. Proposed by W. L. WATSON, Moundsville, West Va.

A solid sector is cut out of a sphere 10 feet in radius by a cone whose vertical angle is 120°. Find the radius of the sphere whose volume is equal to that of the sector.

CALCULUS.

When this issue was made up, solutions had been received for 344-5-6-7, 349, 351, 354, 356, 357, and 358. Solutions of 332, 337, 340, 342, 348, 350, 352, and 353 are desired. A complete solution of 339 is also desired.

359. Proposed by W. D. CAIRNS, Oberlin College.

Examine for maxima and minima

$$f(x) = e^{-cx}(1 + \cos x) \quad (c > 0).$$

360. Proposed by ELMER SCHUYLER, Brooklyn, New York.

What interpretation must be given to

$$\frac{d^{1/2}y}{dx^{1/2}} \text{ so that } \frac{d^{1/2}}{dx^{1/2}}\left(\frac{d^{1/2}y}{dx^{1/2}}\right) = \frac{dy}{dx}?$$

361. Proposed by EMMA M. GIBSON, Drury College.

Determine the system of curves satisfying the differential equation

$$[(1 + x^2)^{1/2} + ny]dx + [(1 + y^2)^{1/2} + nx]dy = 0,$$

and show that the curve which passes through the point $(0, n)$ contains as part of itself the conic

$$x^2 + y^2 + 2xy(1 + n^2)^{1/2} = n^2.$$

(From Forsyth's *Differential Equations*, p. 41.)

362. Proposed by C. N. SCHMALL, New York City.

Having given $y^3 - a^2y + axy - x^3 = 0$, show by Maclaurin's theorem that

$$y = -\frac{x^3}{a^2} - \frac{x^4}{a^3} - \frac{x^5}{a^4} - \cdots.$$

MECHANICS.

When this issue was made up, solutions had been received for 273, 284-5, 288, and 289. Solutions of 266, 268, 269, 271, 274-5, 277, 279, and 286 are desired.

290. Proposed by B. F. FINKEL, Drury College.

A fox, pursued by a hound, is running with uniform velocity over a frail arch in the form of a cycloid; the hound stops at a weak point of the arch, then tumbles through and reaches the level ground with a velocity equal to that of the fox. Prove that the fox exerted no normal pressure on the arch at the point where the hound fell through.

(From Walton's *Problems in Theoretical Mechanics*, p. 605.)

291. Proposed by EMMA M. GIBSON, Drury College.

The time of descent, down a rough inclined plane, of a spherical shell which contains a smooth solid sphere of the same material as itself is t_1. The time of descent, down the same plane, of a solid sphere of the same material and radius as the shell is t_2. Determine the thickness of the shell.

From Loudon's *Elementary Theory of Rigid Dynamics*, p. 188.

292. Proposed by C. N. SCHMALL, New York City.

In a bombardment, a battleship directs its fire at a fort standing on a hill whose height is a feet above the sea level. The angle of elevation of the fort is found to be ϕ. If the initial velocity of the projectile is v, show that the fort will *not* be struck if $v < \sqrt{ag(1 + \csc \phi)}$.

NUMBER THEORY.

When this issue was made up, solutions had been received for 200, 203, 206, 207 and 210. Solutions of 189, 191, 192, 196, 202, 204–5, and 208–9 are desired.

211. Proposed by E. T. BELL, Seattle, Washington.

If an odd perfect number exists, the total number of its divisors is a multiple of 2, but not of 4; or, what is the same thing, an odd perfect number must be of the form $p^{2a-1}n^2$, where p is prime and a is odd.

212. Proposed by ELMER SCHUYLER, Brooklyn, New York.

Given any positive integer N greater than 1; to prove that the sum of all the positive integers less than N and relatively prime to N equals $\frac{1}{2}N \cdot \varphi(N)$.

213. Proposed by R. D. CARMICHAEL, Indiana University.

Prove that no relatively prime integers x and y exist such that the difference of their fourth powers is a cube.

214. Proposed by A. J. KEMPNER, University of Illinois.

Let a be a positive integer $\geqq 2$, and let $T(n)$ denote the number of distinct divisors of the positive integer n, including both 1 and n, so that $T(1) = 1$, $T(2) = 2$, $T(3) = 2$, $T(4) = 3$, ... Show that

$$\Sigma_{n=1}^{n=\infty} T(n)/a^n = \Sigma_{n=1}^{n=\infty} 1/(a^n - 1).$$

The special case $a = 10$ gives, as is easily seen:

$$9\sum_{n=1}^{n=\infty} \frac{T(n)}{10^n} = \frac{1}{1} + \frac{1}{11} + \frac{1}{111} + \frac{1}{1111} + \cdots.$$

SOLUTIONS OF PROBLEMS.

ALGEBRA.

396. Proposed by H. E. TREFETHEN, Colby College.

Show that $1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - \cdots = \sqrt{2}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$.

I. SOLUTION BY HORACE OLSON, Chicago, Illinois.

Let S_1 represent the sum of the series, $x + \dfrac{x^3}{3} - \dfrac{x^5}{5} - \dfrac{x^7}{7} + \dfrac{x^9}{9} + \dfrac{x^{11}}{11} - \cdots$, of which the first member of the proposed equation is a particular case. Let S_2 represent the sum of the series $x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7} + \dfrac{x^9}{9} - \dfrac{x^{11}}{11} + \cdots$. By calculus,

$$S_1 = \int_0^x (1 + x^2 - x^4 - x^6 + x^8 + x^{10} - \cdots)dx = \int_0^x \left(\frac{1 + x^2}{1 + x^4}\right) dx$$

$$= \frac{\tan^{-1}(\sqrt{2}x + 1) + \tan^{-1}(\sqrt{2}x - 1)}{\sqrt{2}}$$